Attorney Docket No. 07-1087 *PATENT*

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re application of: Roger Howard WILLIAMS III

Serial No.: 10/777,586 Group No.: 3692

Filed: February 12, 2004 Examiner: Subramanian, Narayanswamy

For: SYSTEMS AND METHODS FOR IMPLEMENTING AN INTEREST-BEARING INSTRUMENT

DECLARATION OF DR. ARTHUR MAGHAKIAN

I, Arthur Maghakian, declare as follows:

1. I am currently a Managing Director at Natixis. Over the past 10 years, I have held management positions in various credit risk, market risk, and market trading functions. The products that I have developed and/or have risk managed have included: structured credit products, correlation products, hybrid fixed income, equity and equity derivative products.

2. After obtaining my Ph.D. in high energy physics from Rockefeller University, I have published more than 50 articles in refereed journals on topics in theoretical and experimental physics, statistics, and quantitative finance. I am also an inventor named on several U.S. and foreign patent and patent applications for novel Monte Carlo techniques, including U.S. Patents No. 7,149,715 ("Method and System for Simulating Implied Volatility Surfaces for Use in Option Pricing Simulations") and No. 7,228,290 ("Method and System for Simulating Risk Factors in Parametric Models Using Risk Neutral Historical Bootstrapping").

3. My management positions have emphasized: trading management, portfolio optimization, portfolio hedging, and the implementation of integrated market and credit risk frameworks. The bulk of that work has been focused on the development and validation of advanced credit risk models for credit derivatives and structured transactions (including CDO, ABS, CMBS, RMBS, high yield, and corporate bonds, among others).

4. Relative to the risk frameworks that I have designed and implemented, my specific expertise has been focused within the domain of non-linear risk. While a Vice President/Executive Director at Goldman Sachs & Co. from 1995 to 2002, I helped the firm to become the first broker-dealer to win approval from the SEC for the so-called BD-Lite format – a broker-dealer entity designed to optimize capital usage – which resulted in the release of hundreds of millions dollars of capital back to other Goldman Sachs entities.

5. While at Goldman Sachs, I also helped to develop a path-breaking Monte Carlo technique which combined risk-neutral simulation with a historical-simulation approach (patented by Goldman Sachs in the U.S. and elsewhere). This technique permits the ability to work under a risk-neutral measure while preserving observed fat tails and correlation structures (copulas). It also allows one to effortlessly combine cross-market models (e.g. FX and equity, credit spreads and equity volatility, convertible bonds, etc) for the pricing, hedging and trading of hybrid products and strategies.

6. I have also developed a stochastic volatility surface model for application to equity derivatives; that model is used by traders for the hedging of their vega risk. That model was used for both risk management as well as by volatility traders. My quantitative work involved the implementation and calibration of advanced multi-factor models developed within Goldman Sachs for simulating volatility surfaces for equity and FX derivatives, interest rate curves, oil and metal commodities. I have extensive experience in all of these markets, both from the risk and from the trading strategies point of view. My latest publication: "Implications of Stochastic Recovery Rates in Evaluating CDO Tranches" (*Journal of Fixed Income*), focuses specifically on the fixed-income domain. That paper is attached as an Appendix.

Background: Monte Carlo Simulation

7. Monte Carlo Simulation ("MCS") is a universal mathematical method frequently used for solving complex problems applied when direct analytical solutions cannot be obtained. Quickly popularized after invention of computers in 1940s, MSC is a common calculation technique actively used in many research areas and applications, including but not restricted to: applied and theoretical physics, biotechnology and pharmaceutics, space and aircraft engineering, financial engineering, etc.

8. While usable where closed-form solutions to financial/econometric problems are available, one of the considerable advantages to the use of this method is the power and the flexibility that it offers when closed-form solutions are not available. Given this particular advantage, MCS has become a fundamental modeling tool and generally accepted tool with the scientific community at large – MCS finds a number of broad ranging applications – as well as within the domain of modern mathematical finance

9. The essence of the method is the generation of very large number of possible sets of inputs to the analyzed system and observation of the outputs generated by the system for given set of inputs. Even though the inputs are purely random, they follow certain stochastic laws and probability distribution. As a result, statistical properties (e.g. average, standard deviation, etc) of produced outputs converge to the fixed results, no matter what specific random inputs were used

10. Using MCS (e.g., in 7,149,715, as well as in 7,228,290 referenced above) allows calculations to be made with errors which converge, on average, to within any predetermined error bounds

11. In the discussion below, I refer to various points in the flow of the applicant's invention where the an understanding of the pertinence of the use of MCS is paramount

Applicant's Invention

12. Claim 105 describes a computer-based system for processing data regarding an instrument that is structured in a way that permits a free interplay between the factors of principal and rate (or corpus and interest rate strip).

13. The system may be built in a fashion unconstrained by time in a way that allows a free manipulation of the credit risk and extension risk impounded in the instrument. Options in a given market can be made explicit in some formal sense and priced in such a way that the triple of principal, rate, and payment timings are jointly or severally manipulated and under the complete control of the instrument designer and the parties who buy or sell that instrument.

14. It cannot be doubted that such calculations as those that are disclosed by the applicant are among the most complex and computationally "expensive" encountered in modern mathematical finance. Such contingent claims are both computationally demanding and require computations of a type that must be performed rapidly. Relatedly, those computations are generally utilized in markets where prices are evolving rapidly. Using an electronic means to complete the myriad of calculations required to value such an instrument is no mere convenience, but is integral to the accuracy and timeliness required to impute a market-viable price to such instruments for trading purposes.

15. The complexity of such calculations also requires, due to the many steps involved, and the inherent loss of precision in machine arithmetic, a high degree of precision or exactitude in the specific and implementation of: the applicant's invention, any related known routines for calculating cashflow truncation and for simulating the random number streams that represent, *inter alia*, forward interest rates, forward cashflows, or forward volatilities.

16. Relative to all of these considerations, please refer to the above referenced US Patents for a deeper discussion of the methods used in such calculations.

17. Relative to all of these considerations, please refer to applicant's block #40 and blocks #120, #130, #140 and #150 in Figure 1 of the application where such calculations are performed.

18. One key aspect of the invention, claimed in claims 105 and 108, is providing a construction unfettered by time, which allows the control of credit and extension risk.

19. It is clear, certainly in the current environment, that promises to pay a debt impound: substantial credit risk at inception, forward credit risk, and, upon loss, the consideration of the risk of forward recovery. Please refer to my paper attached as an Appendix.

20. As will be discussed below in the points that I make with regard to claims 107 and 109, and with regard to equations #8 and #9 of the applicant's specification – but equally applicable to claims 105 and 108 – it is quite clear that the time parameter may range over any time from the instrument's inception until its liquidation; to the extent that the time period is a domain interval over the real numbers, the choices for extension or curtailment are infinite intervals. Further, the ability to control curtailment or extension risk is entirely within the joint grasp of the debtor and the creditor, with the joint decision to be made being which finite set of numbers should be selected to reach a mutually agreeable and market-clearing decision set of parameters and related prices, volatilities, and rates.

21. In this context, curtailment represents the truncation of cashflows from debtor to creditor, and may result from voluntary or involuntary accommodations between the parties, but is best expressed in a way that allows debtor and creditor to optimize their respective credit postures, with the invention allowing such an accommodation, thus limiting credit risk to both parties in the event of a truncation of cashflows. Furthermore, extension represents, at least in some approximate sense, the inverse of curtailment, not curtailment.

22. In other words, the invention enables receipt of cashflows from an instrument that pays more slowly than would otherwise seem likely or otherwise warranted. One of the chief problems vexing Wall Street at this hour, second only to curtailment and reorganization, is the prospect of payment streams that stretch so far into the future, based on a debtor's inability to refinance, that the resulting cashflows are severely attenuated by the basic concepts embedded within industry-standard time-value-of-money calculations. The invention addresses that problem.

23. Note that, under the law of one price, the debtor and creditor must reach an agreement where that agreement impounds both an agreement upon the terms of the mathematical construction as well as actual action based upon or under that construction. In other words, the invention requires both concrete and tangible actions by both parties based upon deterministic and concrete calculational outcomes.

24. The application explicates current realities: many of the options in the fixed income markets, particularly the mortgage market, are simply not recognized through either an explicit writing or through a proper market calculation and pricing mechanism. A pricing mechanism in accordance with the invention, of necessity, will put

into the hands of the debtor and creditor a method for managing principle, rate, and timing considerations with complete flexibility.

25. As stated in applicant's specification, the current solutions in the fixed income space do not form a "complete market" in the sense normally used in econometrics, or mathematical finance. To that extent, the claimant's invention would be at least an extension, if not completion, of the underlying space and would provide liquidity to the market.

26. That this gapping exists (i.e., the market incompleteness referred to above) is partially recognized by the market's failed attempts to systematically equilibrate convexity differentials between positively and negatively convex instruments. The applicant's invention would allow for a more graceful match between such disparate convexity profiles.

27. Aspects of the invention claimed in claim 107 focus upon an area within which I have done much work and relative to which I have substantial expertise: regulatory capital relief as implemented under Goldman's Broker-Dealer Lite entity, as well as under the Basel-II project that I am focused upon in my current position.

28. Regulatory capital relief is much sought after by the street, and will trade at a premium in the aftermath of the recent market turmoil associated with the sub-prime crisis. This crisis has already led to the write-down of bonds built on the back of mortgage products, thus, the write-down of the underlying instruments should be also be seen as nearly inevitable. While nearly inevitable, there is no clean or smooth method for such an accommodation to be reached between debtors and creditors, absent bankruptcy relief

29. It is indisputable that a product or a structure that would offer such relief would be immensely valuable to both banks and to households in that, having cut their liability exposure, banks and consumers both need a mechanism to cut their asset exposures in a consistent and coherent fashion. The type of mathematical basis swap recommended by the applicant would afford both households and banks an arbitrage-free mechanism for exploiting changes in the basis between rates and the bond, debt or mortgage corpus against which those rates are applied.

30. In examining the claims and specification, including equation #10 at page 27 of the specification, I draw the following conclusions.

31. First, the unamortized loan balance can and will float arbitrarily with interest rates, which rates are, themselves, allowed to be random, but not arbitrary.

32. Second, while rising interest rates may impose the sort of extension risk we currently see in the market, it is clear that applicant's methodology will obviate, or at least palliate, such debt extension. As rates rise, debtors who might otherwise fail to refinance may do so without economic hazard. Conversely, banks are hardly charitable institutions: they will expect to be compensated for their offer of debt relief with a

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commensurate, roughly speaking (i.e., subject to adjusts for transaction frictions or trader "edge"), offset in the rate at which monies were originally loaned.

33. Third, algebraically speaking, the equation "works", by that I mean that:

a. The concept that it articulates is fundamentally sound. Banks should be able to lower exposure, thus lower their capital requirements with the implementation of this product. This reduction is not the *only* mathematically guaranteed result, because it depends upon the unpredictable forward movement of interest rates, but it is an allowed outcome, unlike outcomes for existing products in the market As an allowed outcome, the solution to this equation may be made unique (i.e., can be well-defined and injective in the mathematical senses in which those words are used), thus arbitrage-free, by referring to the forward structure of interest rates as viewed at trade inception

b. From the point of view of mathematical finance, the formula does not contain any mathematical or logical discontinuities. An arbitrage-free structure is created, and neither the debtor nor the creditor gets a "free lunch": each component of the trade is properly priced, or as phrased in option pricing "fairly" priced to model.

c. Once parameterized with input parameters – which may be somewhat arbitrary within bounds of market accepted variation, but which are subject to a market based agreement upon price discovery – the equations are arbitrage-free.

d. The parameters may contain continuous intervals of real numbers, which intervals, by continuity, which would allow a user both a nominally and mathematically infinite range of inputs (e.g., all the decimal represents of the interest rates between 5% and 10% represent an unaccountable infinite set).

e. As a matter of construction, while the allowable input set is infinite, in practice, one completely deterministic set of numbers is chosen as output and agreed upon by both parties to the instrument or contract (this is analogous to a "choice function" where debtor and credit must choose and make exact their intentions).

f. In other words, price discovery causes clustering in the output, which may be inverted into deterministic domain inputs.

g. The instrument created enforces the requirement that exchanges of value take place in a correlated fashion for the duration of the contract.

h. Because future interest rates are, roughly speaking, a coin-toss: 50% of the time interest rates will increase; without claimant's invention, and with a floating rate of interest, the creditor wins and the debtor loses under all such increases; conversely, a fixed rate interchanges the roles of winner and loser.

i. Under a rising rate environment scenario, both the creditor and debtor would save money under claimant's invention. Note that the probability and the value of the prior event (upward move; no invention) is currently 0% and $0 in that no existing product, in an upward rate move, would offer this benefit; the introduction of this product will assign both a probability to that set of events as well as a market enforced value (roughly speaking, the law of one price).

34. Direct consequences of the above are that the debtor and creditor agree and act upon a change of basis; the change of basis improves the credit profile of both the debtor and creditor; as a result of an improvement in the creditor's risk profile, the creditor should experience regulatory capital relief; and as a result of engaging in actions which afford regulatory capital relief, the creditor should be able to offer improved pricing to the debtor.

The October 31, 2007 Office Action

35. The Examiner states, on page 2, that the claims are indefinite for "failing to particularly point out and distinctly claim the subject matter."

36. It is my opinion that those of ordinary skill in the art (i.e., financial engineers skilled in the construction of fixed income instruments, and the variants thereon, as well as conversant with MCS techniques) will be able to precisely bound and describe the applicant's invention from a reading of the specification and the claims. From a personal perspective, the applicant's approach is initially counter-intuitive, but a slightly deeper analysis of the underlying concepts and of the calculations reveals an approach dissimilar from but understandable relative to current market practices and concepts.

37. The Examiner states, on page 2, that "chosen from any possible combination or permutation of principal size and interest rate...makes the scope of the claims unclear."

38. Those skilled in the art will understand, as I have noted above, that modern finance uses MCS extensively. Many problems solved under MCS, at some point in their formulation, are problems in continuous time, which implies infinite sets as inputs, but which localize outputs as deterministic numbers. The implication is that price discovery fixes the solution points.

39. Black Scholes model itself is also associated with integrations over infinite sets, but single numbers are harvested as output. Such calculations are associated with:

values in a distribution, the probability of such values occurring, and the overall expectation formed from the products and summations of such tabulations.

40. For example, with a range of interest rates between 5% and 10%, and with volatilities between 5% and 10%, and with 3 prepayment parameters, one can, for the sake of discussion: form a 5-dimensional Cartesian product in which each element of the resulting 5-dimensional solid is formed from intervals. Under Cantor's continuum hypothesis, each interval represents an uncountable infinity. The solid thus created exists within the bounds of now current market convention, yet: the input choices (the domain points) are infinite; in a like manner, the co-domain is also infinite; and when a choice function is selected by debtor and creditor, the co-domain is reduced to a countable/actionable range. As a matter of market practice, and under risk-neutral arbitrage, as well as under the law of one-price, there will be a much more limited or denumerable set of opportunities that market participants are likely to agree upon as outputs than the infinite sets used as inputs.

41. In other words, using an infinite choice-set as the domain for the applicant's problem does not create an unbounded solution set as the range (even though the co-domain may also be an infinitely addressable 5-dimensional solid). As those of ordinary skill recognize, each market participant will use a choice-function to reduce the co-domain to an agreeably sized personal solution; where any two market participants agree upon the unique solution, tangible actions with concrete consequences will follow.

42. The Examiner asks, on page 2, regarding how the options relate to the instrument, are they "on the interest bearing instrument, or are these options on some other underlying instrument?"

43. It is standard market practice for such instruments to follow a two-phase evolution. In Phase-I, it is highly probable that such instruments will trade as a bundle. In such a case, the market convention would be to treat the entire contingent claim bundle as a unitary object which allows no dissection. In such a case, the options created are an integral part of the instrument. An analogy would be government bonds which traded as a bundle of corpus and coupons before the STRIPS program was launched which allowed the dissection and discretization thereof. In Phase-II, it is probable that the options discussed would no longer be simply embedded within the instrument, but would be stripped and trade freely, much as Treasury bonds were ultimately offered in the STRIPS program. Historically, the US government resisted the idea of stripping bonds, but came to the recognition that a more liquid market would be created thereby.

44. This provides the exact answer: these options are not on some other instrument, and are not – initially – on the underlying instrument, but will be part of the underlying instrument. The options embedded in the interest bearing instrument will be priced as contingent claims on the underlying cashflows of that instrument and on the associated interest rates.

45. The Examiner states, on page 4, "the utility requirement provides that the utility of the invention has to be (i) specific, (ii) substantial and (iii) credible."

46. As discussed above: the invention leads to an algebraically denumerable or identifiable result which is quite specific. The results produced are as fully substantial as the work performed under my own awarded patents. My analysis of the applicant's process is that it is credible in light of my own work and my understanding of the construction and issuance of such instruments.

47. The Examiner states, on page 4, by inference that failure to "produce a real world result", led to a finding that "no substantial practical application" was found.

48. The result, as indicated, would be substantial if implemented, and eminently practical. For instance, as the example provided in the specification at pages 20-22 indicates, a full-life result indicating a value of $12,531 dollars on an indebtedness of $250,000 is roughly 5% of the initial amount at issue.

49. With roughly $8Trn in mortgages outstanding, the deadweight loss to the economy imposed by the incompleteness caused by the market's failure to implement a solution analogous to the applicant's proposed invention may be as high as $400 billion dollars per annum. The avoidance of this decrementality would result in substantial savings for the US economy.

50. The Examiner states, on page 4, "to produce a 'concrete' result, the process must have a result that can be substantially repeatable or the produce must substantially produce the same result again."

51. My assessment is that the applicant's invention results in a real number, and that repeated calculations using the applicant's invention can reproduce identical results, given identical initial conditions (i.e., parameters). The parameters can be varied over a range of the real numbers chosen by a user and the results will be repeatable and predictable.

52. The Examiner states, on page 5, "The limitation of 'any possible combination or permutation of principal size and interest rate' does not produce anything concrete."

53. Please see my comments above: an infinite domain does not presage an infinite range (though the co-domain may be infinite).

54. The fact that the applicant allows infinite flexibility in choice does not mean that decisional closure will not be reached; on the contrary, as those of ordinary skill recognize, such closure is reached via a "choice function" as selected and implemented by the users of the invention.

Summary

55. My prior education, experience, and personal awards of patents in this domain provide me with a substantial body of material on which I may draw to form

opinions regarding applicant's invention. In particular, with specialties in random processes, fixed-income products, and interest rate and volatilities models and surfaces, as well as with respect to regulatory capital, this application impinges directly on the core of my expertise.

56. As noted above, I have reviewed the applicant's application (both specification and claims) thoroughly, including a review of the inputs, the processing, and the outputs.

57. I have found no "holes" or gaps or discontinuities in the logic, the inputs, the processing and the outputs. The process used accords well with my own work in simulations under various types of Monte Carlo process, as well as the variance reduction performed there under.

58. Applicant's invention addresses a void or null space in current market practice. Implemented as disclosed, applicant's invention would fill a practical need and reduce deadweight costs to both debtors and creditors.

59. Applicant's invention appears to be accurate, robust, and concrete. Further, those calculations are flexible enough to accept a variety of parameter ranges as inputs, and to produce a unique and verifiable output.

I believe that the above is true and correct to the best of my knowledge and information.

Maghakian

Arthur Maghakian, Ph.D. Dated: April 29, 2008

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Appendix

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Implications of Stochastic Recovery Rates in Evaluating CDO Tranches

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Institutional Investor

Implications of Stochastic Recovery Rates in Evaluating CDO Tranches

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The binomial expansion technique (BET) originally proposed by Moody's has evolved as a standard technique financial institutions and other market participants use to rate tranches of collateralized bond obligations (CBOs) and collateralized loan obligations (CLOs) (see Cifuentes and O'Connor [1996]). The principal reason for the model's popularity is its simplicity and low implementation cost. The technique is sometimes used to estimate preliminary rating indications prior to developing Monte Carlo simulation models, or for transactions with limited information on the underlying pool of collateral.

BET is reasonably accurate for estimating the ratings of individual tranches if the underlying collateral pools are relatively homogeneous. A multibinomial expansion technique generalizes the original approach, allowing application to non-homogeneous portfolios (see Cifuentes and Wilcox [1998]). Even in the more generalized form, however, the technique relies significantly on the assumption that recovery rates are constant across collateral pools. This assumption is not supported by empirical observation.

Altman and Kishore [1996] show that historical recovery rate density functions for every debt seniority category demonstrate non-zero probabilities across the entire range of recovery rates. The variability of recovery rates should widen the overall distribution of losses for the collateral pool and thus have an impact on expected losses and credit ratings of individual tranches. The impact will vary, depending on the properties of the underlying collateral pool (average rating, debt seniority, time to maturity, and diversity score), as well as on the size of both senior and subordinated tranches.

To analyze the effect of stochastic recoveries on individual ratings, we develop a Monte Carlo simulation model that generates credit events with both constant and stochastic recovery rates. We calibrate the simulation model in constant recovery mode to produce results identical to BET's, and use a random recovery mode to identify the impact of the BET constant recovery assumption.

We analyze a broad spectrum of tranches with different characteristics in order to identify where the impact of varying recovery rates is particularly significant. In addition, we estimate adjustments to constant recovery rates for each case that would bring BET results into agreement with the simulation incorporating non-constant recovery rate assumptions.

I. MOODY'S RATING METHODOLOGY FOR CBO/CLO TRANSACTIONS

Moody's general rating methodology for structured finance transactions is based on the concept of expected loss. An expected loss estimate for each tranche is mapped into the rating using Moody's expected loss table. Depending on the characteristics of the pool of assets, several approaches can be used to produce the portfolio loss distribution. The choice of the appropriate technique is generally driven by the heterogeneity of the portfolio.

A binomial expansion technique (BET) is commonly applied to homogeneous portfolios with a small number of assets (up to approximately 250). For moderately heterogeneous portfolios, an alternative is a multibinomial expansion technique. The lognormal method is an approach that can be applied to homogeneous portfolios with a large number of assets (generally over 500).

These models are the most popular alternatives to the Monte Carlo simulation methodology, along with the Fourier transform method, which is typically used to analyze portfolios of heterogeneous assets (see Debuysscher [2003]). Monte Carlo simulation is generally superior for analyzing all types of portfolios, but it comes at considerable computational cost. The Monte Carlo simulation methodology can also be used to study properties of other models by reproducing their assumptions and assessing the sensitivity of results. Indeed, it is relatively straightforward to build a simulation model that converges toward BET results for a large number of trials.

BET is based on the derivation of a synthetic portfolio of independent and homogeneous assets with identical size, default probability, and maturity and consistent with the average behavior of the underlying pool of assets. The inputs of BET are the diversity score D (a proxy for the number of independent assets that mimic the original pool), the weighted-average maturity of the pool T, the weighted-average default probability of the pool p, and the weighted-average recovery rate of the pool R. The latter parameter—which varies depending on the debt seniority of the underlying assets in the pool—is assumed to be constant.

Using the assumption of independence, one can easily compute the probability P_j of observing j defaults, where j ranges between zero and D, and therefore describes the overall behavior of the synthetic portfolio that reproduces the behavior of the original portfolio:

$$P_j = \frac{D!}{j!(D-j)!} p^j (1-p)^j \qquad (1)$$

where p is the probability of default for a single asset.

The total expected loss is then simply described by:

$$E = \sum_{j=1}^{D} P_j Loss_j \qquad (2)$$

where $Loss_j$ is the loss realized due to j defaults assuming constant recovery rate R.

The expected loss of each tranche of the capital structure depends on two additional parameters: the size of the tranche, and the extent of the subordination. For a tranche with size T and subordination S, the expected loss should be equal to:

$$E_T = \sum_{j=1}^{D} P_j \max[0, \min(T, Loss_j - S)] \qquad (3)$$

The validity of the BET assumptions clearly depends on the properties of the underlying portfolio. Even in the ideal case, when the portfolio is composed of independent and homogeneous assets, additional assumptions may play an important role. These assumptions are related to the stochastic nature of the probabilities of default and recovery rates and to macroeconomic credit cycles. Among these assumptions, the non-variability of recovery rates is the critical one, as the impact of credit cycles may be more or less pronounced, depending on the maturity of a transaction.

II. DISTRIBUTION OF RECOVERY RATES

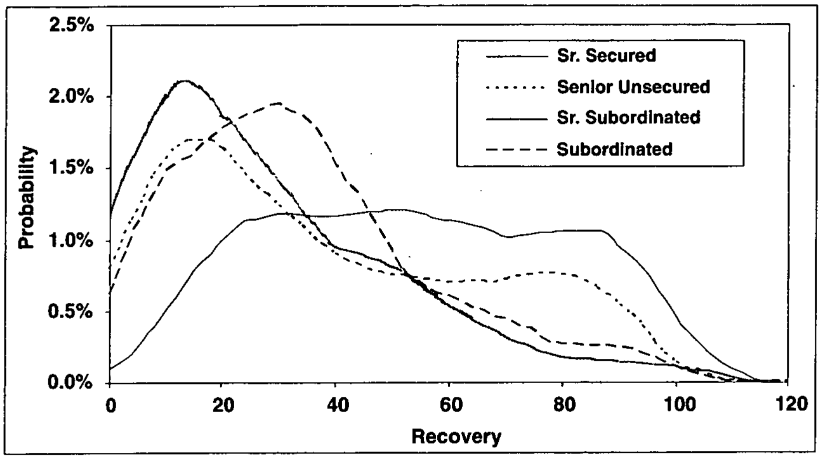
We first define the distribution functions of the recovery rates for each class of debt seniority. Recovery rate is defined as the price of a newly defaulted debt instrument, rather than the price upon emergence from bankruptcy, since the timing and relative success of emergence are generally uncertain.

Altman et al. [2003], Friedman and Sandow [2003], Frye [2003], and Hamilton, Cantor, and Ou [2002] note the direct dependence of recovery rates on economic cycles. In fact, economic downturns appear to have a stronger effect on recovery rates than on default rates. Historical analysis also demonstrates that the impact of economic cycles is more pronounced for the most senior debt classes.

In order to eliminate model dependence from our analysis, we choose not to incorporate dependence on economic cycles in our simulation model, but rely on historical recovery rate distributions published by Moody's for an extended observation period. We do not parame-

EXHIBIT 1

Historical Distributions by Debt Seniority



a function of capital structure and the underlying pool's characteristics to see where there are significant biases produced by the constant recovery rate assumption.

To categorize the characteristics of the pool and the underlying capital structure, we identify parameters including average rating of the underlying pool, seniority of the underlying assets, and size of the tranche and its subordination. In all cases analyzed, time to maturity is fixed at five years, and the diversity score is chosen to equal 25 in order to limit the set of parameters to those that dominate the impact.*

A series of tests are developed to cover a wide range of values for the four parameters. The size of the tranche can range from 5% to 85% and subordination from 5% to 50%. Combining these parameters for the capital structure with four asset seniority types and with ratings ranging from Aa2 to Caa, we construct 134 sample sets of results. When there are rating inconsistencies between the MC and BET models, we estimate the adjustment to the constant recovery rate that would reproduce the ratings based on stochastic recovery rates.

Exhibit 2 demonstrates the results for a set with B2-rated subordinated assets and an 80%/10%/10% capital structure. The results of the MC simulation for each tranche—in terms of expected loss (EL), probability of

terize the empirical distribution for the same reason.

Market practitioners commonly use the beta distribution to describe recovery rate density functions. Despite its several advantages, beta parameterization of historical data would also introduce model dependence and make the results of our analysis less general.

Exhibit 1 shows empirical distributions of recovery rates for four debt categories based on default data over 1970–2003. As would be expected, the means of these distributions are consistent with their seniorities. The curves also demonstrate substantial bimodality, which is the result of the underlying economic cycles. This bimodality is particularly noticeable for senior debt classes, which is consistent with the observation that economic cycles have more severe impacts on senior debt recovery rates. Therefore, the use of historical recovery rate distributions allows us to at least partially incorporate the impact of economic cycles.

III. ANALYSIS AND RESULTS

To analyze the impact of stochastic recovery rates, we implement a Monte Carlo (MC) simulation model constructed around the same assumptions used in BET. The constant recovery rates used by both BET and MC are based on historical averages calculated for each seniority type (e.g., senior debt, subordinated debt).

As expected, the MC simulation, generated with enough trials, accurately replicates BET results for all cases analyzed. The MC model is then modified to incorporate stochastic recovery rates generated according to empirical distributions for each seniority type. Finally, observed differences in expected losses and ratings are analyzed as

EXHIBIT 2

Sample Test Results

		Senior Tranche(s)	Analyzed Tranche	Junior Tranche(s)
	Size	80%	10%	10%
Monte Carlo Simulation	EL	0.002%	3.1%	51%
	ProbLoss	0.1%	12%	89%
	LGD	1.9%	26%	58%
	Rating	Aa1	Ba2	C
Initial BET	EL	0.001%	2.6%	52%
	ProbLoss	0.1%	15%	89%
	LGD	1.7%	17%	59%
	Rating	Aaa	Ba1	C
Adjusted BET	EL	0.002%	3.1%	53%
	ProbLoss	0.1%	15%	89%
	LGD	2.5%	20%	60%
	Rating	Aa1	Ba2	C

EL is expected loss. ProbLoss is probability of experiencing a loss. LGD is expected loss, given non-zero loss.

EXHIBIT 3

Maximum Discrepancy by Debt Seniority

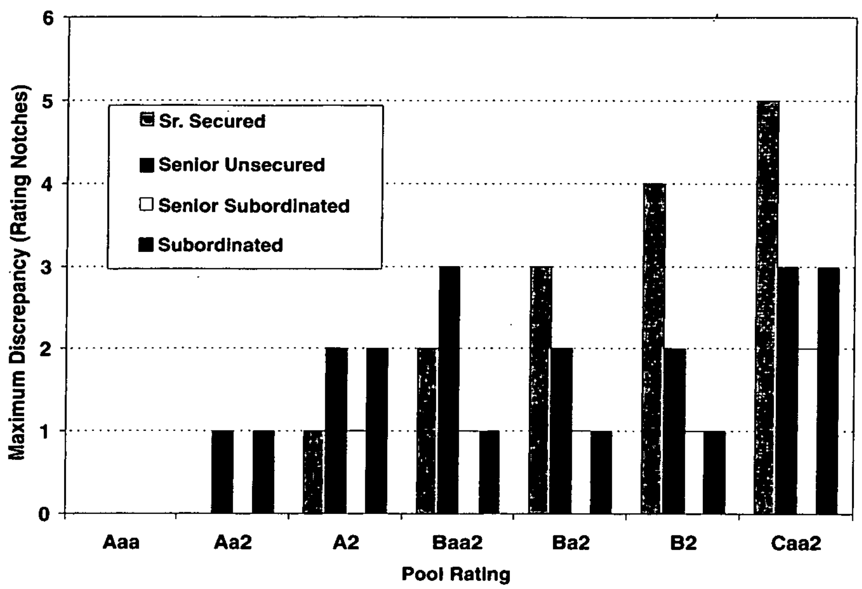


EXHIBIT 4

Average Recovery Adjustment by Average Underlying Rating

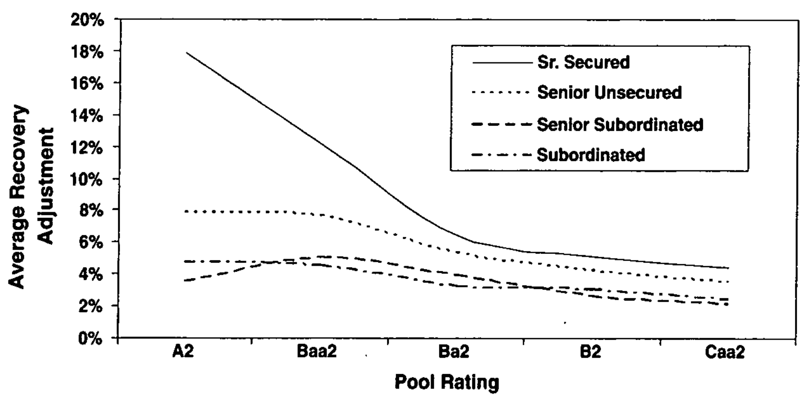
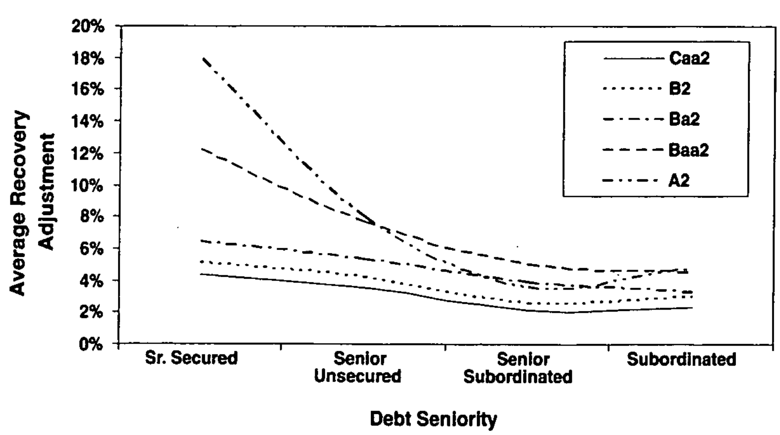


EXHIBIT 5

Average Recovery Adjustment by Debt Seniority



experiencing a loss (ProbLoss), expected loss, given non-zero loss (LGD), and the corresponding rating—are compared to similar results for BET (Initial BET) based on average recovery rates. In this example, we see a one-notch difference for the senior and intermediate tranches.

The last panel displays the results of BET with an adjusted constant recovery rate that would produce the same expected loss for the intermediate tranche. The adjusted constant recovery rate is 32.8% versus the average of 34.9% for subordinated bonds. In this case, the adjusted recovery rate replicates the ratings of both the senior tranche and the intermediate tranche.

Exhibit 3 shows the greatest discrepancies in ratings among analyzed capital structures for a prespecified average credit rating of the pool and debt seniority. The impact of stochastic recovery rates appears to be minimal for high-credit quality pools. For an Aaa pool, observed differences between ratings based on stochastic and constant recovery assumptions are not found to be significant. For non-investment-grade pools, however, the impact is much more pronounced—as much as one to three rating notch differences in ratings for typical capital structures.

Exhibits 4 and 5 display the average recovery adjustments when we see rating discrepancies. The adjustment is defined as the difference between the average recovery for a given type of debt seniority and a modified recovery rate that replicates the expected loss from Monte Carlo simulation.

Exhibit 4 shows that more senior debt requires greater recovery adjustments. Adjustments for pools with senior secured debt as the underlying collateral dominate adjustments for pools with unsecured debt as collateral because of the shape of the distribution curve of recovery rates for senior secured debt. The impact is more pronounced since the senior secured debt recovery rate distribution has almost equal probability weight in the range from 20% to 90%, while unsecured debt has significantly lower averages and narrower distributions that are shifted to the left, with high probability mass around the mean.

Similarly, Exhibit 5 shows that higher-credit quality portfolios require greater adjustments. The steep slope of the A2-rated curve is inconsistent with other curves due to limited underlying statistics, since the adjustment is computed only to match the expected loss for cases with mismatched ratings. For the A2 curve, there are only 2 observations among 134 cases (1.5% of the total) with a difference in ratings. Compare this to 7.5% with differences in ratings for Baa2 and 15% for B2 arrived at using BET.

Exhibits 6 through 10 present differences in ratings and the corresponding adjusted recoveries implied by the use of historical recovery rate distributions for each debt seniority. In the case of a typical capital structure, differences in ratings between standard BET and Monte Carlo simulation range between one and three notches.

IV. CONCLUSION

We have shown that non-constant recovery rates can have a significant impact on BET ratings, particularly for mezzanine tranches of CDOs with non-investment-grade underlying collateral pools. In these cases, more accurate estimates of ratings can be produced by Monte Carlo simulations or by correcting for varying recovery rates.

Similar analysis of other BET assumptions—constant probabilities of default—shows even greater differences in ratings estimated by BET and Monte Carlo simulation when we incorporate changes in the probabilities of default due to macroeconomic and credit cycles. The negative correlation between changes in recovery and default rates may have even greater impact.

Similar adjustments will be required for any other model that relies on a constant recovery rate assumption. Modified constant recovery rates are likely reasonable adjustment proxies for other models.

EXHIBIT 6
A2 Pool Discrepancy and Adjusted Recovery Rates

		Size of Tranche								
Debt Type	Subordination	5%	15%	25%	35%	45%	55%	65%	75%	85%
Sr. Secured (54%)	5%	1 (37%)	1 (36%)	1 (37%)	1 (37%)	1 (37%)	1 (37%)	1 (36%)	-	-
	10%	-	-	-	-	-	-	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Senior Unsecured (39%)	5%	2 (32%)	1 (32%)	-	1 (32%)	1 (32%)	1 (32%)	1 (32%)	1 (32%)	1 (32%)
	10%	-	-	-	-	-	-	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Senior Subordinated (29%)	5%	-	1 (27%)	-	1 (26%)	-	-	-	-	-
	10%	-	-	-	-	-	-	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Subordinated (34%)	5%	2 (30%)	1 (30%)	-	-	-	-	-	-	-
	10%	-	-	-	-	-	-	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				

EXHIBIT 7

Baa2 Pool Discrepancy and Adjusted Recovery Rates

		Size of Tranche								
Debt Type	Subordination	5%	15%	25%	35%	45%	55%	65%	75%	85%
Sr. Secured (54%)	5%	2 (42%)	2 (42%)	1 (42%)	2 (42%)	1 (42%)	1 (42%)	1 (42%)	1 (42%)	1 (42%)
	10%	-	-	-	-	-	-	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Senior Unsecured (39%)	5%	3 (32%)	2 (32%)	2 (32%)	1 (32%)	1 (32%)	2 (32%)	1 (32%)	1 (32%)	1 (32%)
	10%	-	1 (32%)	1 (32%)	1 (32%)	-	-	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Senior Subordinated (29%)	5%	-	1 (27%)	1 (27%)	-	1 (27%)	-	-	1 (27%)	1 (27%)
	10%	-	-	-	-	-	1 (22%)	1 (22%)	1 (23%)	1 (24%)
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Subordinated (34%)	5%	1 (30%)	1 (31%)	1 (31%)	1 (31%)	1 (31%)	1 (31%)	1 (30%)	-	-
	10%	-	-	1 (31%)	1 (30%)	1 (31%)	1 (29%)	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				

EXHIBIT 8

Ba2 Pool Discrepancy and Adjusted Recovery Rates

		Size of Tranche								
Debt Type	Subordination	5%	15%	25%	35%	45%	55%	65%	75%	85%
Sr. Secured (54%)	5%	1 (52%)	1 (51%)	1 (51%)	1 (51%)	-	-	-	1 (51%)	1 (51%)
	10%	2 (48%)	2 (48%)	1 (48%)	2 (48%)	1 (48%)	1 (48%)	2 (48%)	2 (48%)	1 (48%)
	15%	2 (46%)	2 (46%)	1 (46%)	1 (46%)	1 (46%)	1 (46%)	1 (46%)	1 (46%)	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Senior Unsecured (39%)	5%	-	-	-	-	-	-	-	-	1 (36%)
	10%	-	1 (35%)	1 (35%)	1 (35%)	1 (35%)	1 (35%)	1 (35%)	1 (35%)	1 (35%)
	15%	2 (34%)	2 (34%)	2 (34%)	1 (34%)	1 (34%)	2 (34%)	2 (34%)	1 (34%)	
	20%	1 (33%)	-	1 (33%)	1 (33%)	1 (33%)	1 (33%)	1 (32%)	1 (32%)	
	25%	1 (30%)	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Senior Subordinated (29%)	5%	-	-	-	-	1 (29%)	-	-	-	-
	10%	-	1 (28%)	1 (28%)	-	-	-	-	-	-
	15%	1 (25%)	1 (25%)	-	1 (25%)	-	1 (25%)	1 (25%)	1 (25%)	
	20%	1 (26%)	1 (26%)	1 (26%)	-	-	-	-	-	
	25%	-	1 (24%)	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Subordinated (34%)	5%	-	1 (33%)	-	-	-	-	-	-	-
	10%	-	-	-	1 (33%)	1 (33%)	-	1 (33%)	1 (33%)	-
	15%	1 (31%)	-	-	1 (32%)	-	-	1 (32%)	1 (32%)	
	20%	1 (31%)	-	-	-	-	-	1 (29%)	1 (29%)	
	25%	1 (28%)	-	-	-	-	-	-		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				

EXHIBIT 9

B2 Pool Discrepancy and Adjusted Recovery Rates

		Size of Tranche								
Debt Type	Subordination	5%	15%	25%	35%	45%	55%	65%	75%	85%
Sr. Secured (54%)	5%	-	-	-	-	-	-	-	-	-
	10%	-	1 (52%)	-	-	1 (52%)	-	-	1 (52%)	1 (52%)
	15%	2 (51%)	1 (50%)	2 (50%)	1 (50%)	2 (50%)	2 (50%)	2 (50%)	1 (50%)	
	20%	4 (49%)	2 (49%)	2 (48%)	3 (49%)	2 (49%)	2 (49%)	2 (49%)	2 (49%)	
	25%	2 (47%)	1 (47%)	1 (47%)	1 (47%)	1 (47%)	1 (47%)	1 (47%)		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Senior Unsecured (39%)	5%	-	-	-	-	-	-	-	-	-
	10%	-	-	-	-	-	1 (38%)	-	1 (38%)	-
	15%	-	-	1 (37%)	1 (37%)	-	-	1 (37%)	-	
	20%	1 (36%)	1 (36%)	1 (36%)	1 (36%)	2 (36%)	1 (36%)	1 (36%)	1 (36%)	
	25%	2 (35%)	2 (35%)	2 (35%)	1 (35%)	1 (35%)	2 (35%)	1 (35%)		
	30%	2 (34%)	1 (34%)	1 (34%)	-	-	-	-		
	35%	1 (32%)	1 (33%)	1 (32%)	-	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Senior Subordinated (29%)	5%	-	-	-	-	-	-	-	-	-
	10%	-	-	-	1 (29%)	-	-	-	-	-
	15%	-	-	1 (29%)	-	-	-	-	1 (29%)	
	20%	-	-	-	1 (28%)	-	-	-	1 (28%)	
	25%	-	1 (27%)	1 (27%)	1 (28%)	1 (28%)	1 (28%)	-		
	30%	1 (27%)	1 (27%)	1 (26%)	-	1 (26%)	1 (26%)	1 (26%)		
	35%	1 (26%)	1 (26%)	-	-	-	-			
	40%	1 (25%)	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				
Subordinated (34%)	5%	-	-	-	-	-	-	-	-	-
	10%	-	-	-	-	-	-	1 (34%)	-	-
	15%	-	-	-	1 (34%)	-	1 (34%)	-	-	
	20%	1 (33%)	-	1 (33%)	-	1 (33%)	-	1 (33%)	-	
	25%	-	1 (32%)	1 (32%)	1 (32%)	1 (32%)	-	1 (32%)		
	30%	1 (31%)	1 (31%)	1 (31%)	1 (31%)	1 (31%)	1 (31%)	-		
	35%	1 (30%)	1 (32%)	1 (31%)	1 (31%)	-	-			
	40%	-	-	-	-	-	-			
	45%	-	-	-	-	-				
	50%	-	-	-	-	-				

EXHIBIT 10

Caa2 Pool Discrepancy and Adjusted Recovery Rates

		Size of Tranche								
Debt Type	Subordination	5%	15%	25%	35%	45%	55%	65%	75%	85%
Sr. Secured (54%)	5%	-	-	-	-	-	-	-	-	-
	10%	-	-	-	-	-	-	-	-	-
	15%	-	-	-	-	-	-	1 (54%)	-	
	20%	-	-	-	-	-	1 (53%)	-	1 (53%)	
	25%	1 (52%)	1 (52%)	1 (52%)	1 (52%)	-	1 (52%)	1 (52%)		
	30%	3 (51%)	3 (50%)	3 (50%)	3 (50%)	3 (50%)	2 (50%)	3 (50%)		
	35%	5 (49%)	4 (49%)	4 (49%)	3 (49%)	3 (49%)	3 (49%)			
	40%	3 (47%)	2 (47%)	1 (47%)	1 (47%)	1 (47%)	1 (47%)			
	45%	1 (46%)	1 (46%)	-	-	-				
	50%	-	-	-	-	-				
Senior Unsecured (39%)	5%	-	-	-	-	-	-	-	-	-
	10%	-	-	-	-	-	-	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	1 (39%)	-	
	25%	-	-	-	-	-	-	-		
	30%	-	1 (38%)	1 (38%)	1 (38%)	-	1 (38%)	-		
	35%	1 (37%)	1 (37%)	1 (37%)	-	1 (37%)	1 (37%)			
	40%	1 (36%)	1 (36%)	2 (36%)	2 (36%)	2 (36%)	2 (36%)			
	45%	3 (35%)	3 (35%)	2 (35%)	2 (35%)	2 (35%)				
	50%	3 (34%)	2 (34%)	2 (34%)	2 (34%)	1 (34%)				
Senior Subordinated (29%)	5%	-	-	-	-	-	-	-	-	-
	10%	-	-	-	-	-	-	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	-	-	-	-	-	
	25%	-	-	-	-	-	-	1 (29%)		
	30%	-	-	-	-	-	-	-		
	35%	-	-	-	-	-	1 (29%)			
	40%	-	1 (28%)	-	1 (28%)	-	1 (28%)			
	45%	1 (28%)	1 (28%)	1 (28%)	2 (28%)	2 (28%)				
	50%	2 (27%)	2 (27%)	2 (27%)	1 (27%)	2 (27%)				
Subordinated (34%)	5%	-	-	-	-	-	-	-	-	-
	10%	-	-	-	-	-	-	-	-	-
	15%	-	-	-	-	-	-	-	-	
	20%	-	-	-	1 (35%)	-	-	-	-	
	25%	-	-	-	-	-	-	1 (34%)		
	30%	-	-	1 (34%)	-	-	-	-		
	35%	-	1 (34%)	-	-	-	-			
	40%	1 (33%)	1 (33%)	1 (33%)	-	1 (33%)	2 (33%)			
	45%	1 (32%)	2 (32%)	2 (32%)	2 (32%)	1 (32%)				
	50%	3 (32%)	2 (31%)	2 (31%)	2 (31%)	1 (31%)				

ENDNOTE

*Since BET depends on the probability of the default rather than maturity or rating separately, the impact of the probability of the default can be captured through the rating.

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